Motion in a plane

Note : Answers given below are intended only as a reference for the questions asked in board examinations. These answers are NOT an attempt at a detailed discussion related to the question.

VSAQs

1. Define scalar and vector quantities with examples.

Scalar quantities : Quantities which have only magnitude but no direction associated with them are called scalar quantities. Examples : mass, distance, volume, density, electric current etc.

Vector quantities : Quantities which have magnitude, direction and obey vector laws of addition are called vector quantities. Examples : displacement, velocity, acceleration, momentum, force etc.

2. What is a unit vector? Write the unit vectors along the x, y and z – axes.

A vector whose magnitude is unity (i.e. 1) is called a unit vector. Unit vector is used to specify direction.

Unit vector along x – axis : $\hat{i}$

Unit vector along y – axis : $\hat{j}$

Unit vector along z – axis : $\hat{k}$

3. State the conditions under which two vectors are said to be equal.

Two vectors are said to be equal if they have (a) same magnitude (b) they are along same direction

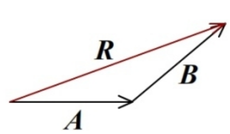
4. What is the physical significance of a null (zero) vector?

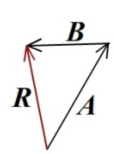
A null vector (**O**) is a vector whose magnitude is zero and whose direction is unspecified. It is significant as it implies that resultant of a mathematical operation between two vectors (addition, subtraction or cross product) yielded a zero.

5. State the triangle law of vector addition.

If two vectors represent two sides of a triangle taken in an order, both in magnitude and direction, then their resultant is given by the closing side of the triangle, both in magnitude and direction, taken in the reverse order.

Example(s)

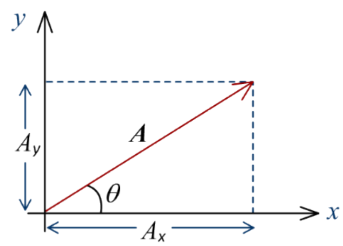




6. What is meant by resolution of a vector?

Resolution of a vector is the process of splitting a single vector into two or more component vectors that produce the same combined effect as the given vector.
Resolution helps us obtain the effect / contribution of a vector in a specific direction.

7. Write the expressions for x and y components of a vector of magnitude A inclined at an angle θ w.r.t. the x – axis.



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$A_x = |A| \cos(\theta)$

$A_y = |A| \sin(\theta)$

8. What is centripetal acceleration? Write its expression in term of angular velocity and radius.

It is the acceleration that is always directed towards the centre of the circular path followed by a body. It is the rate at which an object changes its direction

$$a_{cp} = r\,\omega^2$$

9. State the relation between linear velocity and angular velocity in case of a uniform circular motion.

Linear velocity (v) , angular velocity (ω) and the radius (r) of the circular path are related as

$$v = r\,\omega$$

10. Write the equation for trajectory (path) of a projectile.

$$y = x\tan(\theta) - \frac{g\,x^2}{2u^2\cos(\theta)}$$

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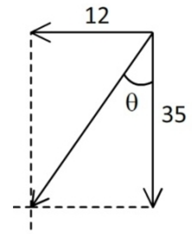
where u is the initial velocity, x & y are the coordinates of the body , θ is the angle of projection and g is acceleration due to gravity

11. Rain drops are falling vertically at 35 ms^{-1} and wind is blowing at 12 ms^{-1} from east to west. In which direction should a person hold an umbrella so that rain drops do not fall on him?

The angle at which the person should hold the umbrella is given the angle made by resultant of velocity of rain drop and the wind velocity



$$\tan(\theta) = \frac{12}{35} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{12}{35}\right)$$

12. Vector is given by $3\hat{i} + 4\hat{j}$. Find the magnitude of the vector and the angle it makes w.r.t. the x – axis.

$$|A| = \sqrt{3^2 + 4^2}$$
$$|A| = \sqrt{25}$$
$$|A| = 5$$
$$\tan(\theta) = \frac{A_y}{A_x}$$
$$\tan(\theta) = \frac{4}{3}$$
$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

13. A projectile is launched with a speed of 20 ms^{-1} at an angle of 30^o w.r.t. the horizontal. Calculate the horizontal component of its velocity.

Horizontal component is given by

$$u_x = u\cos(\theta)$$
$$u_x = 20\cos(30^o)$$
$$u_x = 10\sqrt{3}$$

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14. Two vectors of magnitudes 5 units and 12 units are perpendicular to each other. Find the magnitude of the resultant vector.

$$|R| = \sqrt{A^2 + B^2 + 2AB\cos(\theta)}$$

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$$|R| = \sqrt{5^2 + 12^2 + 2\times 5\times 12\times \cos(90^o)}$$
$$|R| = \sqrt{25 + 144}$$
$$|R| = 13$$

15. A particle's position vector is given by $\boldsymbol{r} = 4t\hat{i} + t^2\hat{j}$. Find its velocity vector at t = 3 s.

$$\boldsymbol{v} = \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\boldsymbol{v} = \frac{\mathrm{d}\left(4t\hat{i} + t^2\hat{j}\right)}{\mathrm{d}t}$$

$$\boldsymbol{v} = 4\frac{\mathrm{d}(t)}{\mathrm{d}t}\hat{i} + \frac{\mathrm{d}\left(t^2\right)}{\mathrm{d}t}\hat{j}$$

$$\boldsymbol{v} = 4\hat{i} + 2t\hat{j}$$

At $t = 2$ s we get

$$\boldsymbol{v} = 4\hat{i} + 4\hat{j}$$

16. A particle moves with constant acceleration $\boldsymbol{a} = 3\hat{i} + 2\hat{j}$ ms^{-2}. If its initial velocity is $4\hat{i}$ ms^{-1}, find its velocity vector at $t = 2$ s.

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}$$

$$\mathrm{d}\boldsymbol{v} = \boldsymbol{a}\,\mathrm{d}t$$

$$\int \mathrm{d}\boldsymbol{v} = \int \boldsymbol{a}\,\mathrm{d}t$$

$$\boldsymbol{v} - \boldsymbol{u} = \int \left(3\hat{i} + 2\hat{j}\right)\mathrm{d}t$$

$$\boldsymbol{v} - 4\hat{i} = 3t\hat{i} + 2t\hat{j}$$

At $t = 2$ s

$$\boldsymbol{v} = 4\hat{i} + 6\hat{i} + 4\hat{j}$$

$$\boldsymbol{v} = 10\hat{i} + 4\hat{j}$$

17. A cricket is ball is hit with an initial velocity of 28 ms^{-1} at 30^{o} above horizontal. What is the time taken by the ball to reach the highest point? (g = 9.8 ms^{-2})

$$TOA = \frac{u\sin(\theta)}{g}$$

$$TOA = \frac{28\sin\left(30^{o}\right)}{9.8}$$

$$TOA \approx 1.4\,\mathrm{s}$$

18. An object is projected such that it reaches a maximum height of 20m. What was its vertical component of initial velocity? (g = 9.8 ms^{-2})

$$H = \frac{u^2\sin^2(\theta)}{2g}$$

$$u^2\sin^2(\theta) = H \times 2g$$

$$u_y{}^2 = 20 \times 2 \times 9.8$$

$$u_y \approx 19.8\ \mathrm{ms}^{-1}$$

19. Find the angular speed of an object completing 5 revolutions in 2 seconds.

$$\omega = \frac{\theta}{t}$$

θ = 5 revolutions = 5 x 2π rad

$$\omega = \frac{10\pi}{2}$$

$$\omega = 5\pi \text{ rad s}^{-1}$$

20. An object is in uniform circular motion with a seed of 10 ms^{-1} along a circular path of radius 5m. Calculate the centripetal acceleration.

$$a_{cp} = \frac{v^2}{r}$$

$$a_{cp} = \frac{10^2}{5}$$

$$a_{cp} = 20 \text{ ms}^{-2}$$

SAQs

1. Explain the analytical method of vector addition and derive the expressions for components of resultant vectors.

Refer to notes **(click here)**

2. State parallelogram law of vectors. Obtain an expression for the magnitude of the resultant of two vectors.

Refer to notes **(click here)**

3. Derive the equations of motion in a plane with constant acceleration using vector notation.

Refer to notes **(click here)**

4. Derive expressions for time of flight and maximum height of a projectile.

Refer to notes **(click here)**

5. What is uniform circular motion? Derive an expression for centripetal acceleration in terms of angular speed.

Refer to notes **(click here)**

6. Describe how velocity and acceleration vectors are represented graphically in a curved motion.

Refer to notes **(click here)**

7. Derive an expression for the horizontal range of a projectile. Show that the range is maximum when the angle of projection is 45°.

Refer to notes (click here)

8. Write the vector form of position, velocity and acceleration of a particle in a plane and explain each term.

Refer to notes (click here)

9. Show that the path followed by a projectile is a parabola.

Refer to notes (click here)

10. Define scalar product. Write it properties.

Refer to notes (click here)

11. Define vector product. Write it properties.

Refer to notes (click here)

12. If | ***a*** + ***b*** | = | ***a*** - ***b*** | then find the angle between ***a*** and ***a***.

$|a+b| = \sqrt{a^2 + b^2 + 2ab\cos(\theta)}$ and $|a-b| = \sqrt{a^2 + b^2 - 2ab\cos(\theta)}$

Using the given condition

$|a+b| = |a-b|$

$\sqrt{a^2 + b^2 + 2ab\cos(\theta)} = \sqrt{a^2 + b^2 - 2ab\cos(\theta)}$

$a^2 + b^2 + 2ab\cos(\theta) = a^2 + b^2 - 2ab\cos(\theta)$

$4ab\cos(\theta) = 0$, a and b cannot be zero therefore

$\cos(\theta) = 0$

$\theta = 90^{o}$